Kirchhoff's Voltage Law

OBJECTIVES

- Define Kirchhoff's Voltage Law
- Discuss how Kirchhoff's Voltage Law applies to Series and Parallel Circuits
- Calculate Voltage drops in a Series and Parallel Circuit

INTRODUCTION

In 1845 as a student Gustav Kirchhoff formulated his circuit laws, later these laws became his doctoral dissertation. Kirchhoff's Voltage Law is sometimes called "Kirchhoff's second law" or "Kirchhoff's loop rule" and is a consequence of the principle of conservation of energy. In this lesson it will be shown how Kirchhoff's Voltage Law applies to a Series circuit and a Parallel circuit.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law helps to solve unknowns when working with electrical circuits. With the use of Ohm's Law and the two Kirchhoff Laws it is possible to solve for all Currents and Voltage drops in a complex circuit.

Some definitions that will help in understanding electronic circuits:

Voltage Rise - Just like it sounds a voltage rise is created by an electrical device in a circuit that creates a Voltage increase or "Rise". Examples of these devices are batteries, solar cells, generators, alternators, and thermocouples. Simply put a Voltage Rise is an increase in electrical pressure. A Voltage Rise may be a positive or a negative.

Voltage Drop - A Voltage Drop occurs when a Current flows through a Resistive electrical component in an electrical circuit. This Voltage drop will be the opposite polarity from the Voltage rise that caused the Current flow, thus named a Voltage Drop. Using Ohm's Law it can be shown that a Voltage is present when Current flows through a Resistor ($V = I \bullet R$). Simply put a Voltage Drop is a decrease in electrical pressure.

Kirchhoff's Voltage Law generally states:

The algebraic sum of all the voltages around a closed path is zero.

Restated as:

The sum of all the Voltage drops around a single closed loop in a circuit is equal to the total source voltage in that loop.

In an electrical circuit, the Voltage across a resistor is always the opposite polarity from the Voltage source. This means if the Voltage source is a positive polarity (+), the Voltage drop across the resistors in that electrical circuit will be a negative polarity (-). If the Voltage source is a negative polarity (-), the Voltage drop across the resistors in the electrical circuit will be a positive polarity (+).

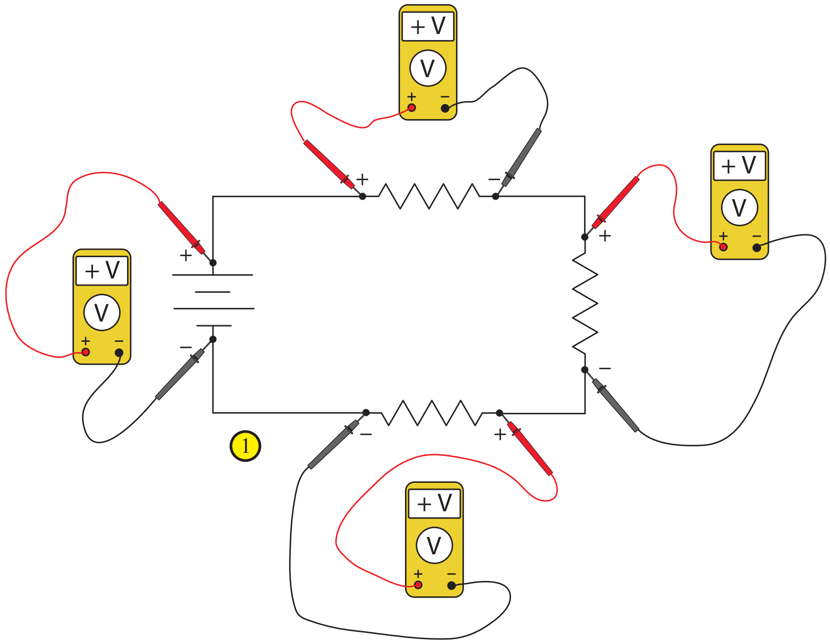


Figure 1

In figure 1, the battery is shown with its polarity marked as a (+) and a (-). The resistors are shown with their true Voltage polarities shown. The multimeters show the Voltage readings following these polarities.

In Figure 1, if you start at the (1) and you look at the polarity indicators (either + or -) follow the electrical circuit clockwise, you will see the - and then the + of the battery, this is followed by each resistors polarity indicators as a + then the - (the resistor's polarity is opposite of the battery's polarity.) It is important to understand that this is exactly what happens in a real electrical circuit.

In Figure 1 again, if you look at the multimeters probes you will note that in this figure the probes are connected with the reds on the positive polarity indicators and the blacks on the negative polarity indicators. This will show a positive reading on all multimeter readings. But this gives the false impression that all Voltages are the same polarity in this circuit (both V_{source} and V_{drops}), this is not the case.

In Figure 2, the multimeters probes are all set in the same direction from (1). If you go counterclockwise the probes are red then black over and over, if you go clockwise the probes are black then red over and over. This will then show the actual Voltage polarity of the V_{source} and V_{drops} if the multimeter probes are referenced to the batteries polarity.

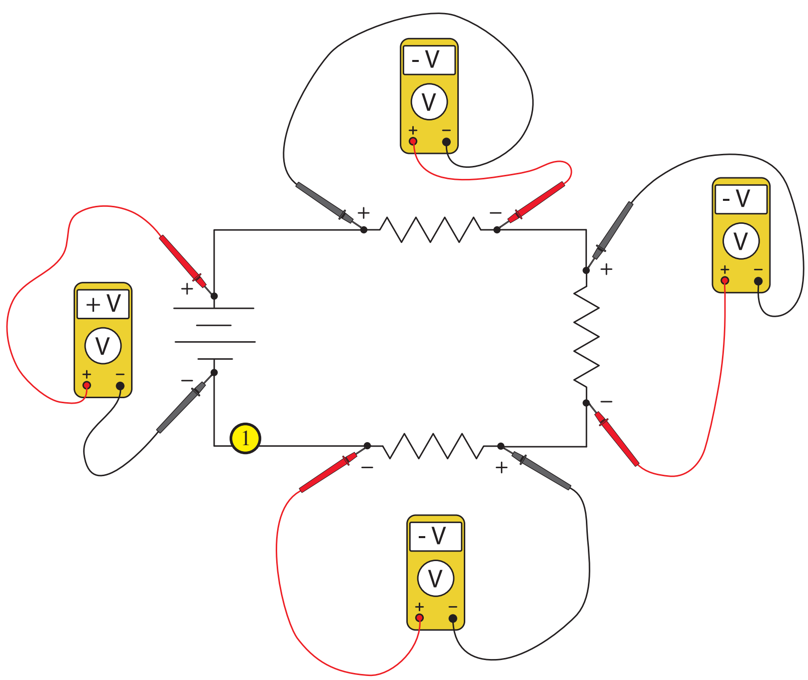


Figure 2

Now we can express Kirchhoff's Voltage Law as an equation.

Kirchhoff's Voltage law expressed algebraically:

The algebraic sum of all the voltages around a closed path is zero.

$$V_{source} + V_{drop1} + V_{drop2} + V_{drop3} + \ldots = 0$$

Example: 12 V + (-3V) + (-4 V) + (-5 V) = 0 V

It's sometimes easier to rewrite the equation to:

The sum of all the Voltage drops around a single closed loop in a circuit is equal to the total source voltage in that loop.

$$V_{source} = V_{drop1} + V_{drop2} + V_{drop3} \ldots$$

Example: 12 V = 3 V + 4 V + 5 V

This equation may look like the Voltage drops have changed from a negative to a positive but that is not the case. What has happened is by moving the Voltage drops to the other side of the equation they had to be subtracted from the Voltage source. Subtracting a negative Voltage drop made them mathematically positive.

Application of Kirchhoff's Voltage Law in a Series circuit

In Figure 3, the battery (B1) is the voltage rise and the resistor (R1) will have the voltage drop across it.

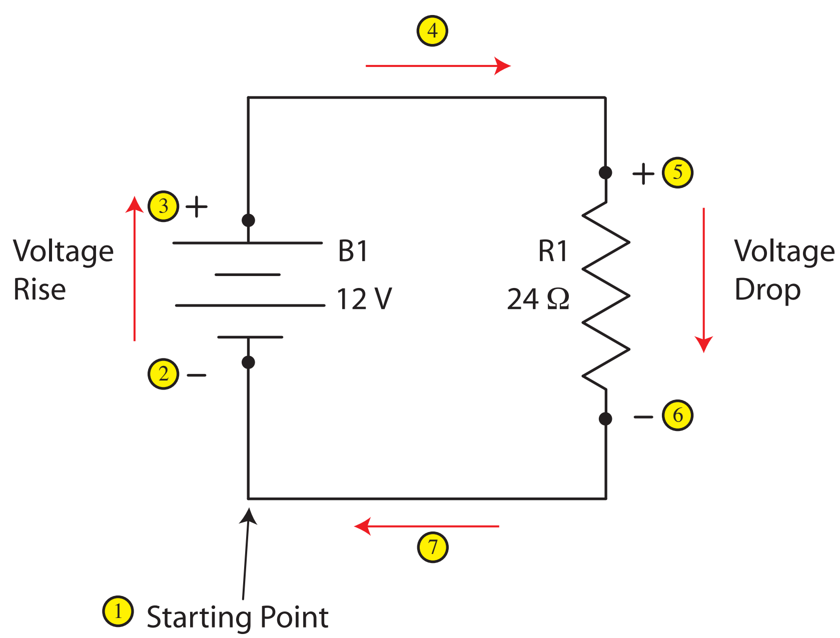


Figure 3

(1) First, we start at a place in the circuit (any point will work) and call that point 0 V.

(2) In this example we will go clockwise around the circuit, but counterclockwise will work also. The first thing we run into is a negative sign on the battery. This means as we go through the battery we will be going more positive than what we started with. (We called that 0 V's remember!)

③ The Voltage has risen 12 Volts from what we started with. (Since that was 0 V, we are now at +12 V.)

④ Until we get to a resistive component the Voltage will stay the same. (+12 V)

⑤ We are now on the positive side of the resistor. As we travel through the resistor, energy will be dissipated as heat and the Voltage will drop.

⑥ At the negative side of the resistor we will have dropped 12 V. (or added a negative 12 V) See Figure 4 for an explanation for a 12 Volt drop across R1 (Ohm's Law review).

Kirchhoff's Voltage Law:

$$V_{source} + V_{drop} = 0\ V$$

$$12\ V + (-\ 12\ V) = 0\ V$$

Vt = 12 V
Rt* = 24 Ω

$$\frac{V}{R} = I \quad \text{So} \quad \frac{12\ V}{24\ \Omega} = .5\ A$$

*R1 is Rt because there is only one resistor.

.5 Amps leaving B1, and .5 Amps returning to B1. So .5 Amps has to be going through R1. (Series circuit).

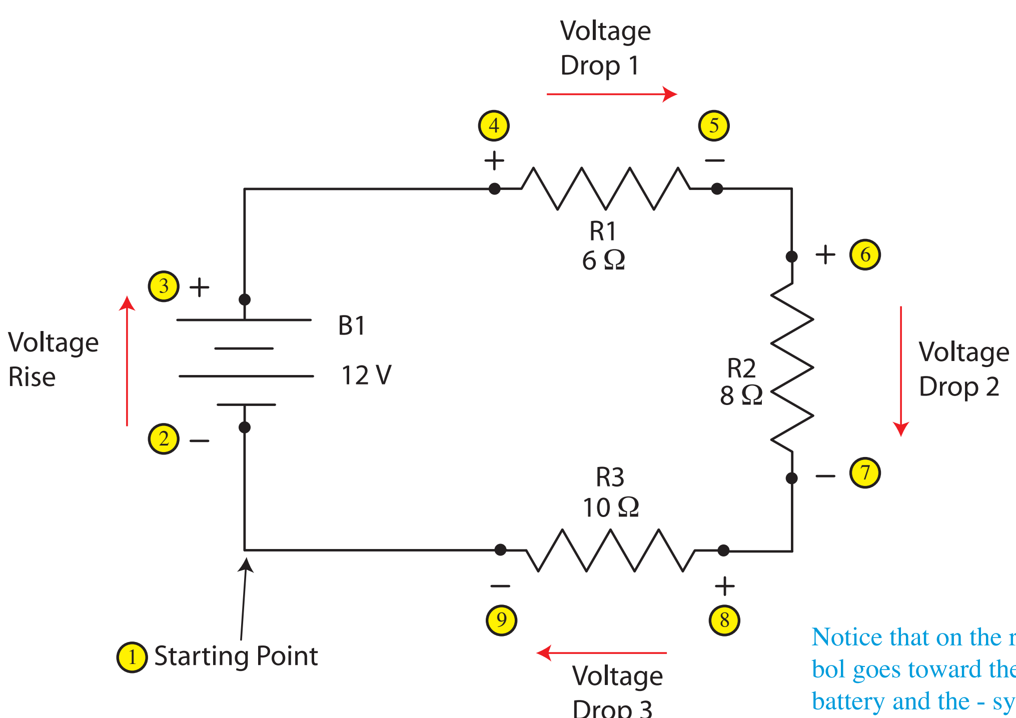
$$I \bullet R = V \qquad .5\ A \bullet 24\ \Omega = 12\ V$$

.5A through 24 Ω equals 12 Volt drop across R1

Figure 4 Review of Ohm's Law

⑦ Until we get to a resistive component the Voltage will stay the same, 0 V. But we are back to the starting point before that and are done. We started at 0 V and finished at 0 V.

In Figure 5, the battery (B1) is the voltage rise and each resistor will have the voltage drop across it.



Notice that on the resistors the + symbol goes toward the + terminal of the battery and the - symbol goes toward the - terminal of the battery.

Figure 5

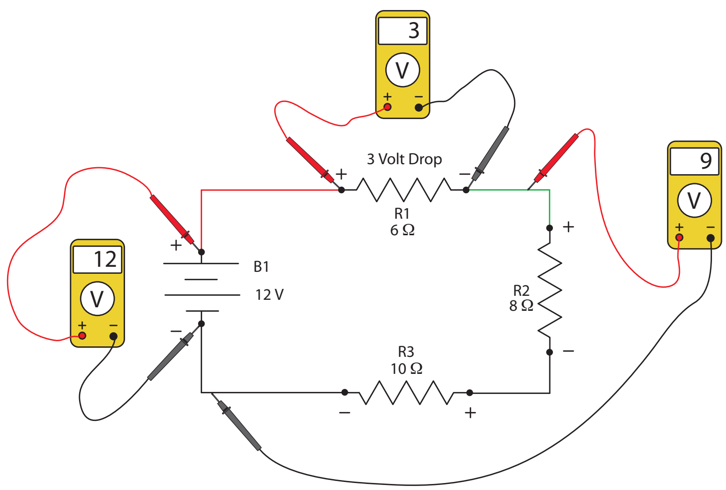
① If we start at a place in the circuit (any point will work) and call that point 0 V.

② In this example we will go clockwise around the circuit, but counterclockwise will work also. The first thing we run into is a negative sign on the battery. This means as we go through the battery we will be going more positive than what we started with. (We called that 0 V's remember!)

③ The Voltage has risen 12 Volts from what we started with. (Since that was 0 V, we are now at +12 V.)

④ We are now on the positive side of resistor 1 (R1), as we travel through the resistor energy will be dissipated as heat and the Voltage will drop.

⑤ At the negative side of resistor 1 (R1), we will have dropped 3 V through R1 leaving 9 V across R2 and R3, (12 V + (-3 V) = 9 V).



In a series circuit if you know the V_{source}, the R_{total} and the resistance you want to find the V_{drop} across, you can use the following equation:

$$\frac{Vs}{Rt} \times R1 = V_{R1}$$

This can be rewritten as:

$$\frac{R1}{Rt} \times Vs = V_{R1}$$

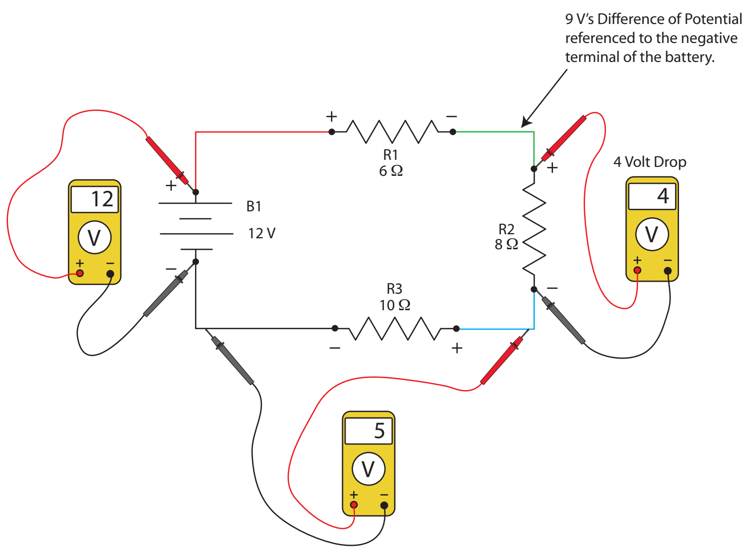
V_{R1} stands for Voltage drop across R1.

Solution for V_{R1}: $$\frac{6\ \Omega}{24\ \Omega} \times 12\ V = 3\ V$$

*Remember Rt for a series circuit equals: Rt = R1 + R2 + R3 . . .

⑥ We are now on the positive side of resistor 2 (R2), as we travel through the resistor energy will be dissipated as heat and the Voltage will drop.

⑦ At the negative side of resistor 2 (R2), we will have dropped 4 V through R2 leaving 5 V across R3, (9 V + (-4 V) = 5 V).



$$\frac{R2}{Rt} \times Vs = V_{R2}$$

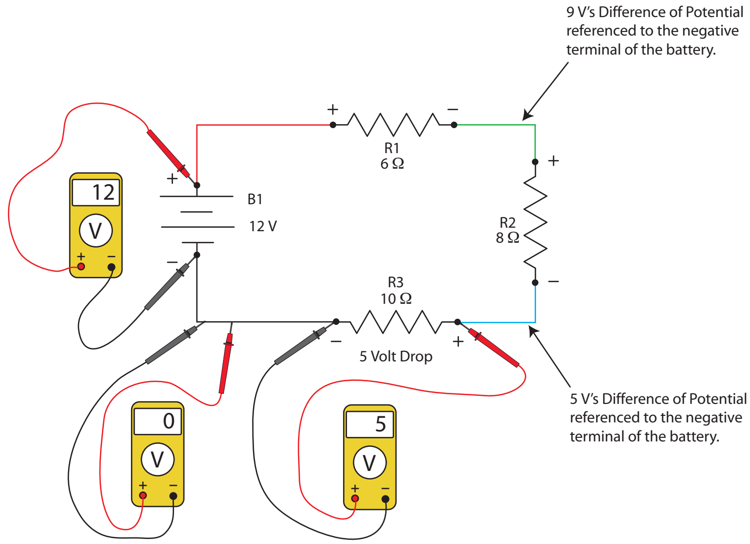
V_{R2} stands for Voltage drop across R2.

Solution for V_{R2}:

$$\frac{8\ \Omega}{24\ \Omega} \times 12\ V = 4\ V$$

⑧ We are now on the positive side of resistor 3 (R3), as we travel through the resistor energy will be dissipated as heat and the Voltage will drop.

⑨ At the negative side of resistor 3 (R3), we will have dropped 5 V though R3 leaving 0 V back to the battery, (5 V + (-5 V) = 0 V).



$$\frac{R3}{Rt} \times Vs = V_{R3}$$

V_{R3} stands for Voltage drop across R3.

Solution for V_{R3}:

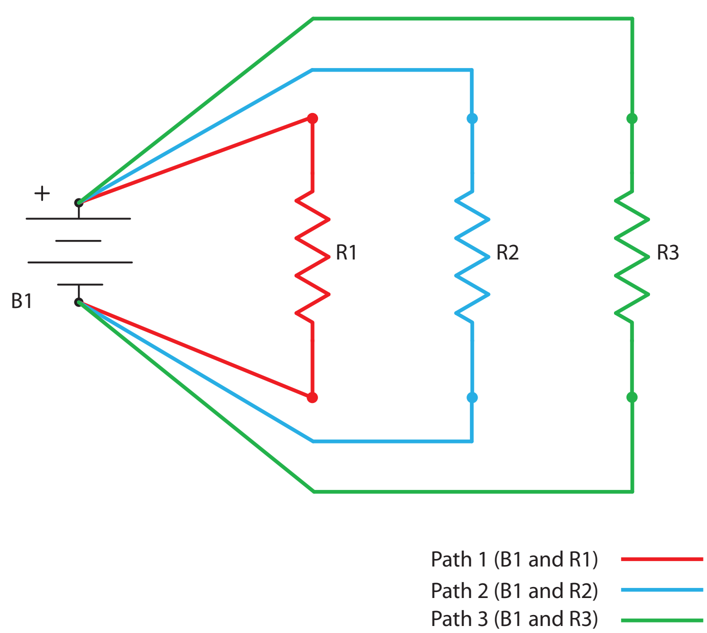
$$\frac{10\ \Omega}{24\ \Omega} \times 12\ V = 5\ V$$

Finally, we started at 0 V and finished at 0 V around the complete "loop" of the series circuit.

In conclusion, no matter how many resistors are in a series circuit, the adding up of all of the Voltage drops together will always add up to the Voltage sources Voltage.

Application of Kirchhoff's Voltage Law in a Parallel circuit

With regard to Kirchhoff's Voltage Law, the important difference between a parallel circuit and a series circuit is that each branch of the parallel circuit will comply with Kirchhoff's Voltage Law individually. Each branch will be handled as a seperate "Loop". See Figure 6 and 6a.



Each path in a parallel circuit is called a branch. There are 3 branches in figure 6, each one will use B1 as the Voltage rise, and each one will have a Voltage drop across it's own resistor.

Figure 6

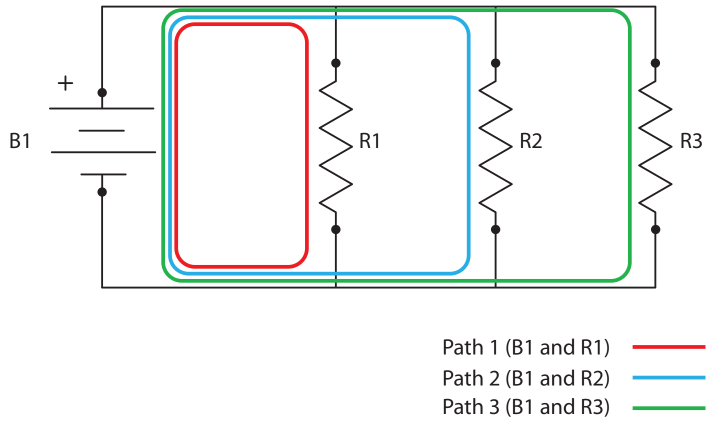


Figure 6a shows a more conventional schematic diagram of a parallel circuit. Figure 6 and 6a are electrically the same. Notice the paths are the same for the branches.

Figure 6a

In figure 6, if the Voltage rise is 12 Volts, R1 will have a Voltage drop of 12 Volts (V_{R1} = 12 V), R2 will have a Voltage drop of 12 Volts (V_{R2} = 12 V), and R3 will have a Voltage drop of 12 Volts (V_{R3} = 12 V). Because the Voltages are all the same it is sometimes said the Voltage is common in a parallel circuit. This can be seen in Figure 7.

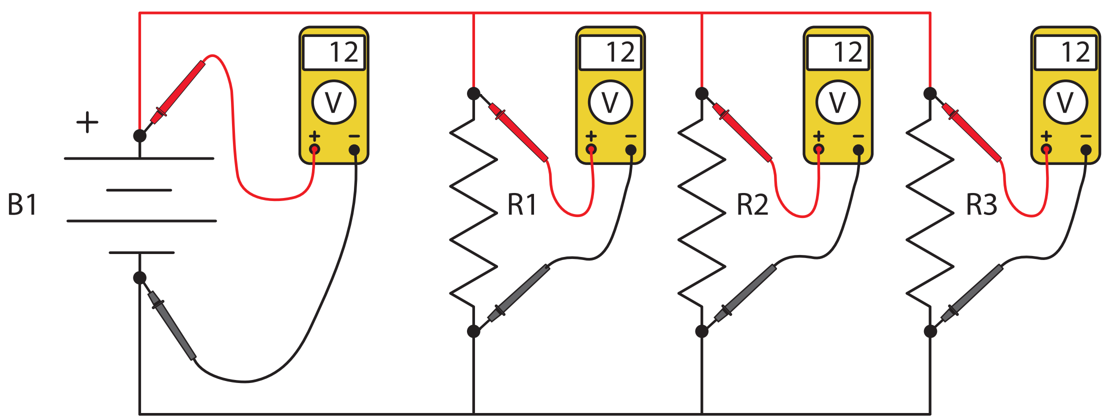
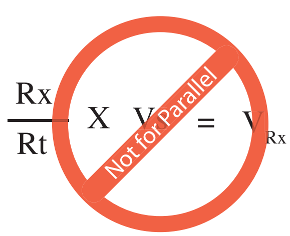


Figure 7

It should be noted that the equation for finding the Voltage drop in a series circuit cannot be used for a parallel circuit. But because the Voltage drops are the same as the Voltage source, an equation is not needed.



For series not parallel.

In figure 8, the circuit is not a basic series circuit nor a basic parallel circuit. It is a combination circuit (part series and part parallel circuit). As you continue with your studies of electronics, it will be found that most circuits are a combination type circuit. This figure is just one example of this type circuit and is meant to get you to think about how Kirchhoff's Voltage Law might apply to this type of circuit.

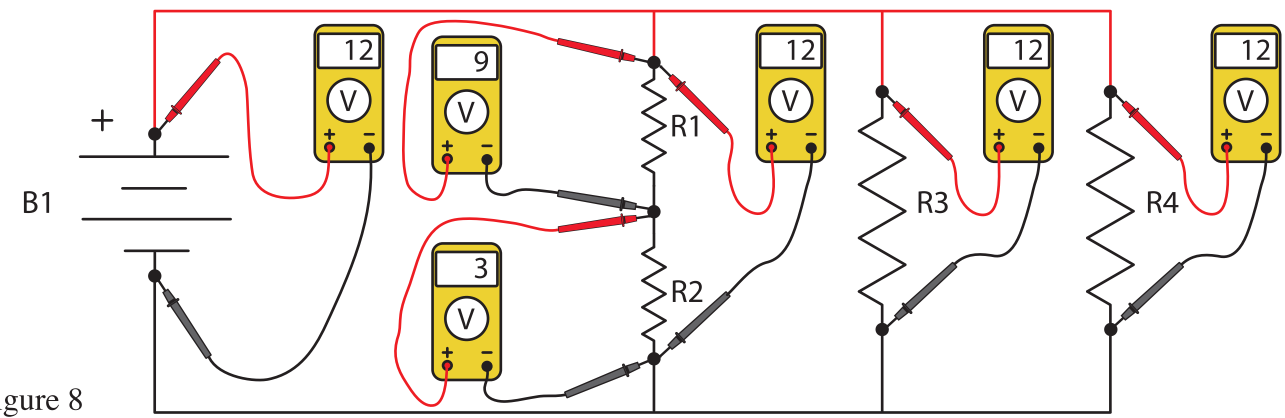


Figure 8

Notice that the Voltage drops for Series resistors R1 and R2 have to add up to the Voltage source. R1 and R2 combined (their resistances added together and thought of as one resistor) are said to be in parallel with B1, R3 and R4. See Figure 9.

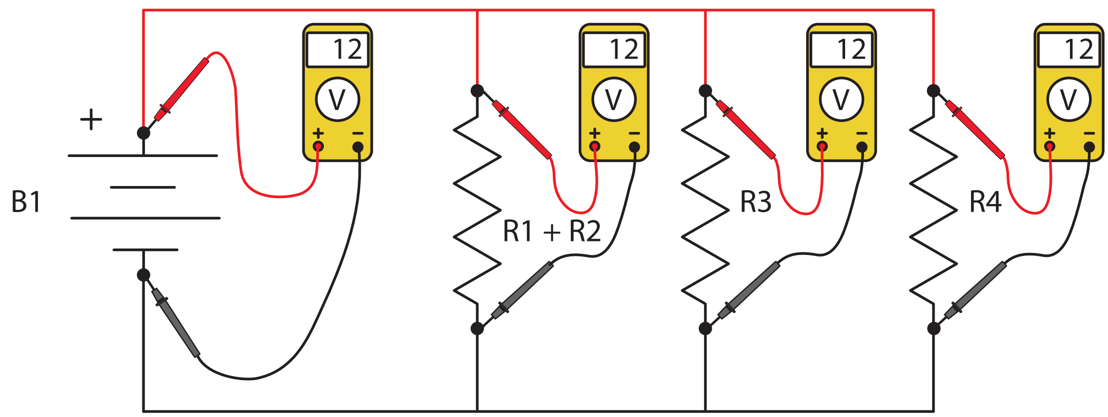
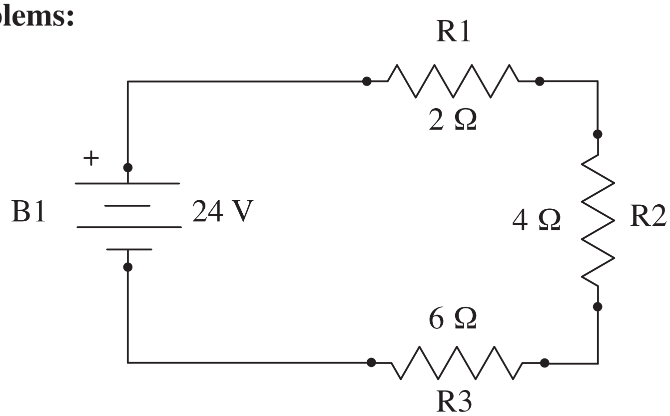


Figure 9

Practice Problems:

1.

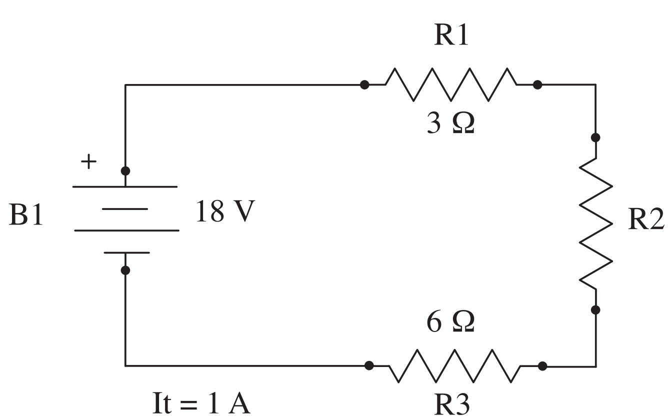


V_{R1} = ____________

V_{R2} = ____________

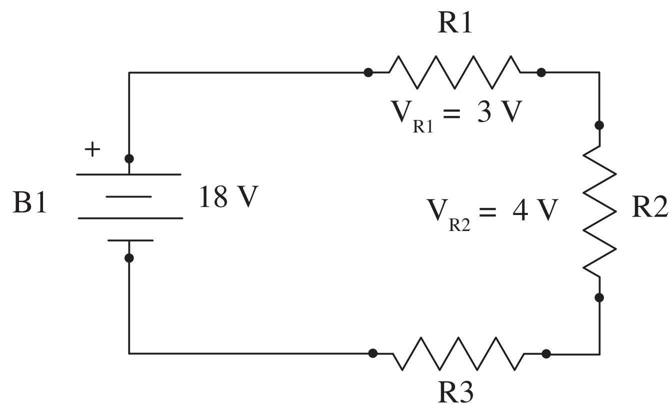
V_{R3} = ____________

2.



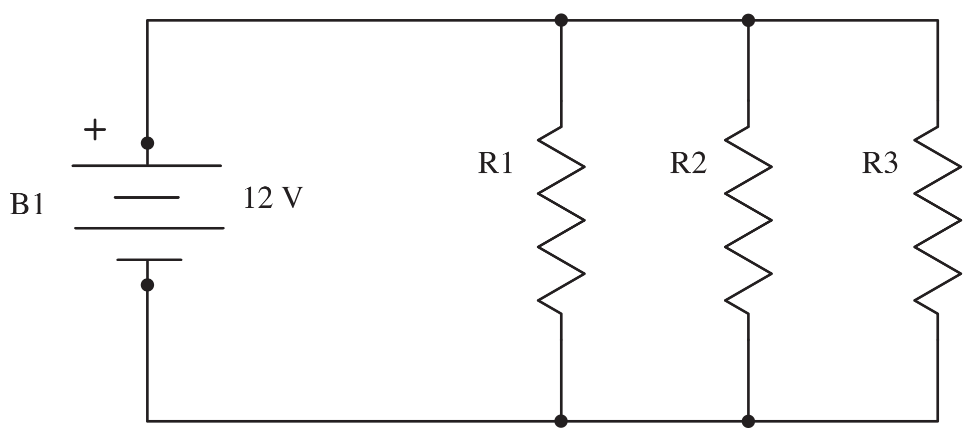
R2 = ____________

3.



V_{R3} = __________

4.

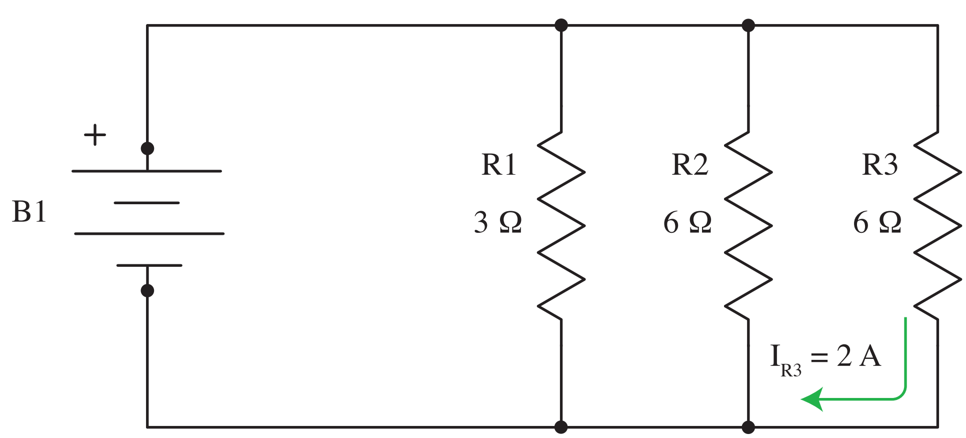


V_{R1} = __________

V_{R2} = __________

V_{R3} = __________

5.



V_t = __________

V_{R1} = __________